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A note on the moduli-induced gravitino problem

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Abstract: The cosmological moduli problem has been recently reconsidered. Papers [1, 2] show that even heavy moduli ($m_\phi > 10^5\,\mathrm{GeV}$) can be a problem for cosmology if a branching ratio of the modulus into gravitini is large. In this paper, we discuss the tachyonic decay of moduli into the Standard Model's degrees of freedom, e.g. Higgs particles, as a resolution to the moduli-induced gravitino problem. Rough estimates on model dependent parameters set a lower bound on the allowed moduli at around $10^8 \sim 10^9\,\mathrm{GeV}$.

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Contents

1. Introduction

The cosmological moduli problem is a disease of many supersymmetry/supergravity theories [3 – 6]. Many supersymmetry/supergravity theories contain fields which have flat potentials in the supersymetric limit and only Planck suppressed couplings to Standard Model (SM) particles. We generically call them moduli. The cosmological moduli problem arises whenever decays of moduli are in conflict with cosmological observations. Masses of moduli depend on the type of supersymmetry breaking. Moduli much lighter than the Hubble scale during inflation acquire a vacuum expectation value (VEV) of order the Planck scale [7, 8] and even exceed it if the mass of moduli is not sufficiently high [9, 10]. In the last case, the modulus field can become an inflaton. Later on, a large abundance of moduli threatens to overclose the Universe or jeopardize the processes of nucleosynthesis. Several solutions of the moduli problem have been suggested, see e.g. [11 – 14].

The cosmological moduli problem is automatically avoided in heavy moduli scenarios. A widely used estimate for the perturbative decay rate $\Gamma_{\rm all}$ of moduli is

$$\Gamma_{\rm all} \;\sim\; \frac{1}{4\pi}\frac{m_\phi^3}{M_p^2}\,. \tag{1.1}$$

where ϕ is the modulus field and m_ϕ is the modulus mass. Moduli decay once the Hubble rate is of the order of $\Gamma_{\rm all}$. Therefore, moduli of mass below 100 TeV decay near or after the time of nucleosynthesis, when the universe is nearly 1 second old. If the mass is above 100 TeV then the moduli decay before the time of Big Bang Nucleosynthesis (BBN). Examples of scenarios with heavy moduli exist [15 – 19].

The heavy moduli scenario as a solution of the cosmological moduli problem has recently been reconsidered starting with the papers [1, 2]. It was shown that the decay of moduli into gravitinos is unsuppressed (for an opposite example see [20]). The part of the Lagrangian describing the gravitino-modulus couplings is

$$e^{-1}\mathcal{L} = -\frac{1}{8}\epsilon^{\mu\nu\rho\sigma}(G_\phi\partial_\rho\phi + G_{\phi^\dagger}\partial_\rho\phi^\dagger)\bar{\psi}_\mu\gamma_\nu\psi_\sigma - \frac{1}{8}e^{G/2}(G_\phi\phi + G_{\phi^\dagger}\phi^\dagger)\bar{\psi}_\mu[\gamma^\mu,\gamma^\nu]\psi_\nu \tag{1.2}$$

where ψ_μ stands for the gravitino and G_ϕ is a non vanishing dimensionless auxiliary field with $G = K/M_p^2 + \ln(|W|^2/M_p^6)$. The subscript i denotes the derivative with respect to the field i. K and W are Kähler potential and superpotential respectively. Based on these coupling, the perturbative decay rate of moduli into gravitinos is

$$\Gamma_{3/2} \equiv \Gamma(\phi \to 2\psi_{3/2}) \approx \frac{|G_\phi|^2}{288\pi} \frac{m_\phi^5}{m_{3/2}^2 M_p^2} \,. \tag{1.3}$$

The auxiliary field of the modulus, G_ϕ, in general, can be small to suppress $\Gamma_{3/2}$ to the total decay rate $\Gamma_{\rm all}$ (1.1). However, suppressed G_ϕ is not a typical case, e.g. in the framework of the $4D$ supergravity $G_\phi \gtrsim m_{3/2}/m_\phi$. Performing elaborate calculations, authors of [1, 2] have shown that the typical branching ratio $Br(\phi \to 2\psi_{3/2}) \sim \mathcal{O}(0.01 - 1)$. The large branching ratio of heavy moduli into gravitinos causes gravitino overproduction. Hence, even having a modulus mass above 100 TeV does not resolve the cosmological moduli problem. A detailed re-analysis of the cosmological moduli problem taking into account constraints on gravitino overproduction pushes up the gravitino mass above $10^5 - 10^6$ GeV [2]. This is the moduli-induced gravitino problem.

The previously published literature on the moduli-induced gravitino problem does not include nonperturbative decay channels. We propose a solution of the moduli-induced gravitino problem by having most of the moduli energy decay into the SM degrees of freedom through a tachyonic decay into a boson pair, e.g. Higgs. The decay process *moduli* $->$ *bosons* is rapid and occurs before moduli start to perturbatively decay into gravitinos. The scheme allows to find a range of scalar moduli masses ($m_\phi > 10^8 \sim 10^9$ GeV) which does not suffer from the moduli-induced gravitino problem. Making use of conservative approximations, we find a range of masses with no overproduction of gravitinos.

2. Basic idea

The general idea can be introduced in the following way. As was mentioned previously, moduli have only Planck suppressed couplings to other fields and during inflation obtain a VEV of the order of the Planck scale. After inflation, the modulus field slowly rolls preserving its energy. When the Hubble parameter reaches the value of m_ϕ, the modulus field starts to oscillate. In the following, we assume that moduli have a trilinear coupling to a scalar field χ,

$$\phi\chi^2 \,. \tag{2.1}$$

The effective potential, $V(\phi, \chi)$ is

$$V(\phi,\chi) = \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}m_\chi^2\chi^2 + \frac{1}{2}\frac{\alpha}{M_p}m_\phi^2\phi\chi^2 + \frac{1}{4}\lambda\chi^4 \,. \tag{2.2}$$

The equation of motion for χ field with switched off the expansion of space is

$$\ddot{\chi}_k + \left(k^2 + m_{\rm eff}^2\right)\chi_k \; = \; 0 \,. \tag{2.3}$$

where

$$m_{\rm eff}^2 = m_\chi^2 + \lambda\chi^2 + \frac{\alpha}{M_p}m_\phi^2\phi \,, \tag{2.4}$$

The oscillations of the field ϕ induce a negative mass for the field χ. The modes of the field χ with $k < \sqrt{-m_{\rm eff}^2}$ are excited,

$$\chi_k \propto e^{\sqrt{-m_{\rm eff}^2 - k^2}t} \tag{2.5}$$

and the energy is transferred from the oscillating ϕ into excitations of χ in a preheating-like process. The process has a name of tachyonic resonance or tachyonic (p)reheating and is widely discussed in the literature starting with [21–23], in particular, the implementation of tachyonic resonance in the context of the resolution of the moduli problem is discussed in [14]. Thus, we see that for a certain range of parameters, the energy density stored in the moduli nonperturbatively transfers into excitations of χ field much before moduli perturbatively decay into gravitinos. The couplings of χ to Standard Model particles are assumed to be unsuppressed and, as a result, the decay rate of χ is much larger than 1 $\sec^{-1}$. Thus, the modulus energy is converted into radiation much before the time of BBN.

To study the stability of the potential (2.2), we find the minimum of the $V(\phi, \chi)$ in the ϕ direction which occurs for

$$\phi = -\frac{1}{2}\frac{\alpha}{M_p}\chi^2 \,. \tag{2.6}$$

Substituting (2.6) into $V(\phi, \chi)$ leads to

$$V(\phi, \chi) = -\frac{1}{4}\left(\frac{1}{2}\frac{\alpha^2}{M_p^2}m_\phi^2 - \lambda\right)\chi^4 + \frac{1}{2}m_\chi^2\chi^2 \,, \tag{2.7}$$

and, we see that the effective potential is unstable for

$$\frac{1}{2}\frac{\alpha^2}{M_p^2}m_\phi^2 > \lambda \,. \tag{2.8}$$

Thus, the presence of additional terms with Planck suppressed couplings is important to stabilize the potential (2.2) at large values of the fields.

The efficiency of the tachyonic resonance must be carefully checked against the effects of dilution due to the expansion of space. For the tachyonic resonance to be effective, the growth of the mode k (2.5) shall dominate the dilution due to the expansion of space. The appropriate condition would be

$$\sqrt{-m_{\rm eff}^2 - k^2} > H \tag{2.9}$$

or

$$\frac{\alpha}{M_p}m_\phi^2\Phi > \frac{m_\phi^2\Phi^2}{M_p^2} \tag{2.10}$$

where Φ is the amplitude of the ϕ field. In the last step, we replaced H with the appropriate contribution from the modulus field. Further, we make an assumption that the energy density of the modulus is the significant component of the total energy density. If this is not the case, the moduli-induced gravitino problem disappears. The reason is that the produced

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gravitino represent only small portion of the total energy density. The condition (2.10) is fulfilled once

$$\alpha M_p > \Phi\,. \tag{2.11}$$

At the onset of oscillations $\Phi < M_p$, thus for $\alpha \geq 1$ we can neglect the expansion of space in our analysis.

In addition to the growing mode (2.5), there is also the decaying mode

$$\chi_k \propto e^{-\sqrt{-m_{\rm eff}^2 - k^2}t}\,. \tag{2.12}$$

The decaying mode causes inference terms and may put further restrictions on the region of applicability of the tachyonic resonance. The equation (2.3) can take the form of the well known Mathieu equation (see e.g. [24]). In fact as it can be seen from the instability chart of the Mathieu equation, the resonant production is terminated as soon as $q \equiv \alpha\Phi/M_p \leq 1/2$; hence we are interested only in cases with $\alpha \gg 1$. Models where α has to be smaller than 1 can be of interest if many trilinear interactions enhance the resonant effect. The condition on α is the same as in (2.11) which means that the resonance production is efficient once the change in the scale factor is negligible.

Tachyonic preheating in the parameter range corresponding to large α was extensively studied in [25]. The authors have shown that trilinear terms lead to faster re-scattering and thermalization. As a bonus, trilinear terms allow complete decay of the moduli. In addition to positive effects, enhanced resonance and fast subsequent thermalization may enlarge the reheating temperature beyond the allowed region which threatens to overproduce gravitinos through re-scattering processes [26].

The trilinear interaction term (2.1) may arise, for example, from the non-renormilizable term in the Kähler potential [1]

$$\mathcal{L}_H = \int d^4\theta \frac{\lambda_H}{M_p} \phi H_u^* H_d^* + h.c. \tag{2.13}$$

where H_u and H_d are up-type and down-type Higgs supermultiplets or corresponding scalar fields, respectively. The ϕ field is the moduli supermultiplet and, in the following, its scalar part. After integrating out the superspace coordinates, we obtain

$$\mathcal{L}_H = \frac{\lambda}{M_p}\left(D_\mu D^\mu \phi H_u^* H_d^* + F_\phi H_u^* F_d^* + F_\phi H_d^* F_u^* + c.c. + \cdots\right)$$

where $F_i = -M_p^2 e^{G/2}(G^{-1})^i_j G_j$ is the auxiliary field of the i'th supermultiplet, D_μ is the covariant derivative. The process of energy transfer described above makes use of on-shell degrees of freedom. Hence, we make use of the equation of motion for the ϕ field to replace $D_\mu D^\mu \phi$ with $m_\phi^2 \phi$. As a result, the following interaction term is part of the Lagrangian:

$$\mathcal{L}_H \supset \frac{\lambda}{M_p} m_\phi^2 \phi H_u^* H_d^* + h.c. \tag{2.14}$$

[1] Here we provide only one example of the origin of trilinear terms. Large α might require other interactions.

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In the low energy effective Lagrangian, the term (2.14) is responsible for the interaction (2.1), where χ is the neutral scalar component of the lightest Higgs field in the mass basis.

3. Estimates

In the following we would like to estimate the region of moduli mass for which the moduli-induced gravitino problem is resolved. Another glance at the equation of motion of the χ field

$$\ddot{\chi}_k + \left(k^2 + m_\chi^2 + \lambda\chi^2 + \frac{\alpha}{M_p} m_\phi^2 \phi\right) \chi_k \;=\; 0\,, \tag{3.1}$$

reveals that the tachyonic process is more effective for larger masses of the moduli. We assume that the tachyonic resonance works as long as $m_{\rm eff}^2$ can obtain negative values,

$$\frac{m_\chi^2}{m_\phi^2} < \alpha \frac{\Phi}{M_p}\,. \tag{3.2}$$

All the energy converted into excitations of the χ field afterwards is transferred to SM degrees of freedom. Further, since $Br_{3/2} = \mathcal{O}(0.01 \sim 1)$ we assume that once the bound (3.2) is violated all the energy is transferred to gravitinos. The above assumptions allow us to estimate the gravitino abundance neglecting the effect of the expansion of space. At the end, we insert the known bounds on the gravitino abundance and derive the lower bound on the gravitino mass.

We distinguish between two cases at the onset of moduli field oscillations: in the first case, the universe is supercooled and $\langle\chi^2\rangle \sim 0$; or, in the second case, the universe is dominated by radiation and $\langle\chi^2\rangle \sim T^2 = \sqrt{m_\phi M_p}$. The universe is supercooled if oscillations of the moduli were preceded by an inflationary period, and the energy is still stored in the oscillations of an inflaton, or if the modulus itself is the inflaton (see [27, 28] for discussions on the moduli-induced gravitino problem in this case). In this paper, we primary concentrate on the first case. In this case, we omit the self interaction term to obtain order of magnitude estimates for the bound on the allowed moduli mass.

While the tachyonic resonance is in effect, the energy density in ϕ is transferred to χ particles and then to radiation. Neglecting the expansion of space,

$$\rho_{\rm rad} = m_\phi^2 M_p^2 \tag{3.3}$$

The tachyonic resonance ends as soon as Φ reaches the value

$$\Phi_{\rm min} = \frac{m_\chi^2}{m_\phi^2}\frac{M_p}{\alpha}\,. \tag{3.4}$$

At this point, the remaining energy density in the moduli is

$$m_\phi^2 \Phi_{\rm min}^2 = \frac{m_\chi^4 M_p^2}{\alpha^2 m_\phi^2} \equiv \rho_{3/2}\,. \tag{3.5}$$

The energy density stored in the gravitino, $\rho_{3/2}$, allows us to determine the gravitino abundance.

$$m_{3/2}Y_{3/2} \equiv m_{3/2}\frac{n_{3/2}}{s} \tag{3.6}$$
$$= \frac{\rho_{3/2}}{s} \tag{3.7}$$
$$= \frac{m_\chi^4 M_p^2}{\alpha^2 m_\phi^2 s} \tag{3.8}$$

where $Y_{3/2}$ is the gravitino yield, $n_{3/2}$ is the number density of gravitino particles and s is the entropy of the ultra-relativistic particles.

$$s = \frac{\rho + p}{T_R} = \frac{4}{3}\frac{\rho_{\rm rad}}{T_R} \approx (m_\phi M_p)^{3/2}\,, \tag{3.9}$$

where T_R is the reheating temperature (temperature of ultra-relativistic plasma at the moment it reaches thermal equilibrium). While the actual reheating temperature depends on the thermalization processes, the upper bound is

$$T_R < \sqrt{m_\phi \Phi_{\rm in}} \leq \sqrt{m_\phi M_p} \tag{3.10}$$

where $\Phi_{\rm in}$ is the amplitude of the field ϕ at the onset of oscillations. Since we have neglected the expansion of space throughout the calculations, we have plugged $T_R = \sqrt{m_\phi M_p}$ to obtain the last equality in (3.9).

The gravitino abundance is severely constrained in order not to jeopardize the success of BBN or from the danger of overproducing of lightest supersymmetric particles. The most stringent constraint comes from the overproduction of ^3He [29, 30] which yields

$$m_{3/2}Y_{3/2} < O(10^{-14} \sim 10^{-11})\ \text{GeV}\,. \tag{3.11}$$

The limit (3.11) is equivalent to

$$\begin{aligned} m_{3/2}Y_{3/2} &= \frac{m_\chi^4}{\alpha^2 m_\phi^4} T_R \\ &= \frac{3}{4}\frac{m_\chi^4}{\alpha^2 m_\phi^4}\sqrt{m_\phi M_p} \\ &< O(10^{-14} \sim 10^{-11})\ \text{GeV} \end{aligned} \tag{3.12}$$

where we have inserted the expression for s (3.9). Making further assumptions: $\alpha \sim O(1)$, $m_\chi \approx 100\,\text{GeV}$, the moduli is safe from the overproduction of gravitinos in direct decay if

$$10^8 \sim 10^9\ \text{GeV} \leq m_\phi\,. \tag{3.13}$$

The lower bound (3.13) is the main result of the paper.

In the second case, when the field χ is a part of the thermal bath and the contribution of the self interaction term to the effective mass can be large, we have

$$\begin{aligned} m_{\rm eff}^2 &= m_\chi^2 + \lambda\langle\chi^2\rangle + \frac{\alpha}{M_p}m_\phi^2\phi \\ &= m_\chi^2 + \lambda T^2 + \frac{\alpha}{M_p}m_\phi^2\phi\,, \end{aligned} \tag{3.14}$$

where we have used the Hartree approximation to go from the first to the second line. The large λT^2 term threatens to prevent the tachyonic resonance from occurring. Particulary, if, at the onset of oscillations, the condition

$$1 < \frac{\alpha}{\lambda}\frac{m_\phi}{M_p} \tag{3.15}$$

is not satisfied, the effective mass (3.14) is positive. In an expanding moduli-dominated universe, the temperature redshifts as

$$T^2 = m_\phi M_p \left(\frac{\Phi}{M_p}\right)^{4/3} \tag{3.16}$$

Hence, m^2_{eff} remains positive during oscillations of the ϕ if

$$1 > \frac{\alpha^3}{\lambda^3}\frac{m_\phi}{M_p} \tag{3.17}$$

where we have inserted $\Phi_f = \frac{m^2_\phi}{M_p}$ - the value of Φ at the time of perturbative decay (1.1). In the case $m^2_\chi > \lambda T^2$, the estimates on moduli mass reduce to (3.3)–(3.13).

The decay of moduli dilutes the pre-existing abundance of gravitinos. Let us denote the initial gravitino yield by $Y_{3/2}$. The entropy produced in the decay of moduli into radiation $s_n \propto T^3_n$, hence, the new gravitino yield is

$$\begin{aligned} Y^n_{3/2} &= \frac{n_{3/2}}{s_f + s_n} Y^n_{3/2} \approx \frac{Y_{3/2}s_f}{s_n} = \frac{Y_{3/2}s_f}{s_n} \\ &= \frac{T^3_f}{T^3_n} Y_{3/2}\,. \end{aligned} \tag{3.18}$$

where s_f and T_f stands for the values of the preexisting entropy and temperature of radiation at $\Gamma_{\text{all}} = H$. Making use of (3.16), we deduce

$$Y^n_{3/2} = \frac{m_\phi}{M_p} Y_{3/2} \tag{3.19}$$

4. Conclusions

In this paper, we have discussed the influence of the tachyonic resonance on the moduli-induced gravitino problem. We primarily have discussed the case when χ is not a part of the thermal bath at the onset of oscillations of the modulus field which is a main contributor to the total energy density. In this case, the rough estimates shows that moduli masses above $10^8 \sim 10^9$ are free from overproduction of gravitinos in direct decay of moduli. The estimates omit several model dependent points which may either enhance or diminish the influence of the resonance. In particular, in the process of calculations we did not take into account the expansion of space. In the case when χ is a part of the thermal bath at the onset of the oscillations of ϕ, we have found that the tachyonic resonance is less likely to work. In any case, even if the tachyonic resonance is inefficient, the decay of moduli

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dilutes the preexisting abundance of gravitinos. If the energy density of the moduli is sufficiently subdominant to the total energy density, the moduli-induced gravitino problem disappears. The reason is that the produced gravitino represent only small portion of the total energy density.

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